Abstracts

June 3, 2024

Lecture 1 (10:00 – 11:00)

Sabourou Saitoh

Emeritus Professor of Gunma University, Japan

Division by Zero 1/0 = 0/0 = 0 and Division by Zero Calculus; Their Basic Properties and Impacts

In this presentation, we introduce division by zero

$$1/0=0/0=0,$$

division by zero calculus:

$$\frac{f(x)}{x^n}(x=\mathbf{0}):=\frac{f^{(n)}(0)}{n!};$$

$$tan(\pi/\mathbf{2})=\mathbf{0},$$

$$[z^n/n]_{n=\mathbf{0}}=logz$$

and the new information on real.div in computers.

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June 3, 2024

Session 1 (12:00 – 13:30)

Katalin Munkacsy

Eötvös Loránd University, Hungary

Kelecsényi Klára

University Neumann János, Hungary

Teaching analysis in secondary schools and in non-mathematical higher education

Bressoud revealed serious problems in the teaching of analysis worldwide. Nevertheless, there are no signs of fundamental change. We have found two types of answers. There are those who want to abolish the teaching of analysis for all those who are not preparing to become mathematicians or who would not choose other fields requiring very strong mathematics. Instead, they would teach data science or, for example, probability. Others would follow the tradition of teaching analysis that has developed over the last 100 years, by trying to make each step of the process as accessible as possible.

We have raised the possibility of a major change and would like to present the details of this change. We draw on new methods developed in other areas of mathematics, such as the work of the van Hiele couple in teaching geometry, or Zoltán Dienes' abstract algebra games.

Mónica Arnal-Palacián

University of Zaragoza, Spain

Identification of the mathematical elements involved in the answer on the infinite limit of a sequence by prospective teachers

Teaching and learning of limits has been a subject of study in the field of mathematics education, where in recent years, studies focused on prospective teachers have garnered particular interest. For this reason, two objectives are established in the present study: 1) to analyse the resolution and mathematical elements involved when prospective teachers perform the limit at infinity of a sequence; 2) to characterize the professional perspective of prospective teachers based on the evaluation of different responses presenting various resolutions of the limit at infinity of a sequence. To achieve these objectives, an instrument with two tasks, one for each objective, is used, developed from modifications of a previous instrument that served as a pilot test. The first task gathers information on the calculation

of 3 sequence limits, closely related to their content knowledge. In the second task, 5 real student responses are presented, where prospective teachers must identify the mathematical elements and strategies used, interpret student understanding, and decide on actions to help the student progress in comprehension. The sample was composed of 15 students from the Master's program in Secondary Education Teaching at a Spanish university. Among the results of the first part of the study are both algorithmic procedures and others where underlying mathematical elements allow for limit resolution, employing different representation frameworks. From the results of the second task, it is inferred that future teachers identify some mathematical elements, such as the use of sequences, functions, specific notation, handling of infinity signs, but they do not adequately account for some linked mathematical processes, neither basic nor advanced thinking, nor the approach used, whether intuitive or formal.

Barbara Barańska

University of the National Education Commission, Kraków, Poland

The equation of a tangent line

I remember this moment when, as a student, I learned in one of my math classes the equation of a line tangent to a graph of a function at a point. Neither I nor my classmates had any idea why this equation had such a bizarre form. We did not understand where it originated from, and we had considerable trouble with memorizing the equation. Understanding this equation came years later. However, this personal experience from years ago is still very important to me and inspired me to analyse Polish textbooks to see how contemporary authors introduce the equation of a line tangent to the graph of a function at a point.

June 3, 2024

Session 2 (15:30 – 17:00)

Mateusz Tofilski

Szkoła w Chmurze, University of Silesia in Katowice

Development of Mathematical Skills in Self-Directed Learning and Homeschooling - Analysis of Educational Tools in Szkoła w Chmurze

Homeschooling is on the rise in Poland, especially in secondary school, where student numbers surged by 369% between 2021 and 2023 (Fundacja Edukacji Domowej, 2023). Szkoła w Chmurze (School in the Cloud) is the largest homeschooling institution in the country, with thousands of students. Apart from providing an organized community, it offers students access to an educational platform aimed at facilitating the fulfilment of the core curriculum. The platform is designed to be a friendly ecosystem of remote learning, in which the student is provided with a set of tools to develop their knowledge and competencies. These tools are based on the capabilities of modern technologies and oriented towards practicality. The student is an active user of the platform, who, gets opportunities for its independent exploration of the particular topic. Internal surveys indicate that the School's students show a high sense of agency in their education and see an improvement in their learning competencies after joining the School.

The paper will consist of an analysis of the educational tools available at the School during mathematics learning, with a focus on the basic assumptions behind this educational environment and its assumed impact on the development of key competencies. The paper will point out both the advantages and limitations of this unconventional approach to teaching mathematics.

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mathematical TRANSGRESSIONS

Deepika Bansal, Venkateswaran Thathamangalam Viswanathan

Indian Institute of Science Education and Research Mohali, India

Points of intersections in middle school science and mathematics in Indian curriculum – identifying opportunities for integrated teaching learning

World over, under the STEAM, scholars are championing transdisciplinary/ multidisciplinary teaching-learning to replace the disciplinary silos in the current educational model. The new education policy 2023 of India aims to make school and college education more holistic, multidisciplinary, and flexible. During grades 6, 7, and 8, the middle stage, the policy states that experiential learning within each subject and exploring relations among different subjects will be encouraged and emphasised despite introducing more specialised subjects and subject teachers. The secondary stage, grades 9, 10, 11 and 12, will comprise four years of multidisciplinary study. The middle stage envisages introducing a multidisciplinary approach, which will be built in subsequent stages.

The importance of mathematics and mathematical skills to learning of science is well known. Mathematics and mathematical modelling are deeply woven into modern science and are integral and essential features. Recent scholarship highlights that science can provide meaningful context for introducing and building mathematical concepts and help make mathematics education more meaningful. Teaching and learning integrated mathematics and science could enrich and enhance students' understanding of both disciplines.

In India, school science education includes mathematics, primarily in physics, but it is often reduced to 'problem-solving', applying the relevant formula and numerical values in consistent units. The deep structures encoded by mathematics are lost. Further, many opportunities that lend themselves to mathematical forms and representations are avoided, failing to build students' competency to make meaning in science.

Conversely, school mathematics in India is 'dull' and far removed from reality. Students are clueless about the ritual they are commanded to perform, such as operations or algorithmic steps. Drawing mathematical concepts from substantive real-life situations and skills from applications of mathematical concepts to real-life problems would help the conceptual development of mathematical and scientific reasoning. School mathematics can be made more relevant by using contexts in science rather than abstract, manufactured situations.

A more integrated approach would enable 'transfer' between the disciplines, which enhances pupil engagement and improves students' scientific and mathematical understanding.

India comprises 28 states and eight union territories. As school education is a concurrent subject, each state has its educational set-up, with distinct curricula, textbooks, etc. However, the National Council for Educational Research and Training (NCERT), a union government institution, makes the national curriculum framework and prepares a set of textbooks, which are used in schools following central board of school education standards. In this paper, we examine the middle-stage science textbooks to

identify the points of integration of school science and school mathematics, thereby identifying contexts of science that can be used in teaching mathematical concepts and skills required in learning science.

Collaboration between the science and mathematics curriculum writers is a viable mode to create more meaningfully integrated teaching-learning materials. Our study will contribute to shaping multidisciplinary teaching-learning materials for the NCERT curriculum.

Jessica Lins de Souza Fernandes

Federal University of Santa Catarina, Brazil

Lives in the Philosophy: Samba Schools' parades as an Ethno-Mathema-Tics

The Ethnomathematics Program (EP) is a research programme with a special interest in investigating arts and techniques (tics) of explaining and knowing (mathema) in cultural contexts (ethno) that have historically endured attempts to regulate them. Brazilian Samba Schools, in turn, are cultural associations that emerged as a popular response to institutional racism during a historical period when the State apparatus persecuted the black population and its various manifestations. As an expression of the Brazilian Black Movement, they are educational spaces that produce emancipatory knowledge and teach the public about Brazilian black history and culture, along with their struggles. To achieve this, they use parades as their main communication tool with society, telling stories through special floats, costumes, props, and samba-plot. Using diatopical hermeneutics, we establish a dialogue between the categories of parades and Ethnomathematics through seven diatopical meaning and interpretation keys. Six of them come from the dimensions of the EP as a General Theory of Knowledge: Historical, Conceptual, Cognitive, Epistemological, Political, and Educational. In addition, we analyse the Methodological approaches that underpin EP and Samba Schools' practices. We point out that the parades constitute an Ethno-Mathema-Tics, and that Samba Schools and the Ethnomathematics Program intersect in seven paths of emancipation: i) methodologically, they intersect in the dimension of respect; ii) historically, in strengthening roots; iii) conceptually, in creativity; iv) cognitively, in solidarity; v) epistemologically, in reinvention; vi) politically, in resistance strategies; and finally and initially, vii) from an educational perspective, they find their common path in struggles for emancipation.

mathematical TRANSGRESSIONS

June 3, 2024

Lecture 2 (18:00 – 19:00)

Norman Wildberger

University of New South Wales, Sydney, Australia

Towards a logical rational arithmetic as a foundation for maths education and research

Descartes and Fermat in the 17th century introduced an arithmetical approach to geometry, allowing for the application of algebra to formerly purely geometrical questions. This initiated the "analytic approach" to curves and subsequently to the Calculus and is now the standard way of starting many pure mathematical topics. The preferred arena for this modern approach is that of the "real numbers": however these are never really adequately presented, and a clear derivation of their key properties is a gaping logical hole in our subject. Nevertheless, Calculus, Differential Geometry, Algebraic Geometry and much of modern Number theory are supposedly built from this "foundation".

Modern computer systems are however not as gullible as undergraduate students, and it is clear that they are not capable of incorporating "real number arithmetic" in a computationally explicit way. So we have an increasing divergence between what pure mathematics researchers and educators say — and what their computers can do. "We work over the real numbers!" yet … most of our examples are over the integers or the rational numbers, with a few token "sqrt(2)"s and "pi"s thrown in.

How to transgress from this false orthodoxy? The key is to reassert the primacy of integer arithmetic and its offspring rational number arithmetic. But can we do geometry, and Calculus, and number theory etc. in this framework? Yes we can. Let's see how this Pythagorean program can be revitalized for modern times.

June 3, 2024

Session 3 (19:00 – 19:30)

Łucja Farnik

University of the National Education Commission, Poland

Selected manifestations of Lakatos' quasi-empiricism in contemporary algebraic geometry

Karl Popper, in his famous work "The Logic of Scientific Discovery" (1934), formulated methodological principles for empirical sciences. The methodology of mathematics proposed by Imre Lakatos in his work "Proofs and refutations" (1963-64, 1976) was an attempt to show that the development of mathematics is governed by quasi-empirical rules analogous to the rules of the development of empirical sciences. According to Lakatos, in mathematics the method of proofs and refutations does not only apply to "visible" or "nearly empirical" objects, such as polyhedra, but is also present in the study of theoretical objects, such as sequences. In the view of Lakatos, mathematics exhibits fallibility in a way comparable to the fallibility of natural sciences in Popper's philosophy.

My talk is an attempt to answer the question whether research in a certain area of contemporary algebraic geometry initiated by the Hilbert's 14th Problem is quasi-empirical in the sense of Lakatos. At the end some general methodological conclusions regarding the practice of research in mathematics will be presented.

mathematical TRANSGRESSIONS

June 4, 2024

Session 4 (10:00 – 12:00)

Eva Ulbrich[1], **Mathias Tejera**[1], **Angelika Schmid**[2], **Zsolt Lavicza**[1]

[1] *School of Education, Johannes Kepler University, Linz, Austria* [2] *University of Ostrava, Czech Republic*

From STEM to STEAM: mathematics teachers creating 3DMP projects for their lessons

Traditionally, mathematics lessons are perceived by students as challenging and disconnected from their personal lives and not creative. Creativity, alongside critical thinking, communication and collaboration, is part of the four Cs of the 21st century and is identified as crucial to foster (Trilling & Fadel, 2012). Such skills can be promoted by introducing art and design thinking into STEM education creating STEAM learning experiences (Wittayakhom & Piriyasurawong, 2020). Using technology in STEAM education can offer many advantages such as connecting to students' lives out of schools and fostering the four C's of child development (Trilling & Fadel, 2012). Technologies such as 3D modelling and 3D printing (3DMP) gain popularity for STEAM education and can offer opportunities to support developing the four C's of the 21st century.

However, teachers sometimes hesitate using emerging technologies such as 3DMP which seem appropriate for an integrated STEAM teaching approach. Preparing teachers to use technology can be crucial to using technologies appropriately in schools (Tondeur et al., 2019) so we developed a non-compulsory university course in Austria aiming at assisting teachers in training to use 3DMP for their lessons. Over 100 Austrian students studying two subjects, one of them mathematics and another STEAM subject, aged 22 to 46, all before or at the beginning of their professional careers, took part in this course. They had to create 3DMP projects for their lessons documenting their ideas, challenges and results.

To learn which challenges teachers overcame and whether 3DMP led them to create STEAM projects we looked at these project documentations categorising the project ideas by focusing on mathematics, multidisciplinary approaches and STEAM projects. While some of the projects focused mainly on mathematics, most of the projects were at least multidisciplinary. Many contained STEAM ideas aiming at fostering creativity which led us to conclude that 3DMP not only in itself is a STEAM activity but also inspires mathematics teachers to create STEAM learning experiences.

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mathematical TRANSGRESSIONS

Cecilia Russo, Zsolt Lavicza

Johannes Kepler University, Austria

Fostering Motivation and Transparent Assessment: Integrating Game Elements in Education

As math teachers, we have perceived students' lack of motivation, particularly for doing mathematical exercises and being active during lessons. Additionally, numerous studies (Gottfried et al., 2001; Jacobs et al., 2002) underscore the gradual decline in students' intrinsic motivation as they progress through their academic journey. This decline can be attributed to various factors, such as the emergence of diverse interests, increasing complexity of mathematics, teaching practices, and low levels of enjoyment. Some researchers, such as Lim and Champan (2015), have concluded that motivation is one of the most impactful domains that exhibit a relation with accomplishment in mathematics. Moreover, traditional assessment methods often lack transparency, leaving students uncertain about teachers' expectations, personal progress, and areas for improvement.

Both motivation and assessment are closely linked to game use in education, as Yu et al. (2020) have summarised. In this context, we can distinguish two main concepts: serious games and gamification. While serious games refer to games "designed to convey learning material in being played through" (Deterding et al., 2011, pp. 10), gamification is defined as "the use of game design elements in non-game contexts" (Deterding et al., 2011, pp. 10). In summary, gamification utilises game elements to enhance the experience in a non-game-related activity, whereas serious games are whole games designed for specific purposes beyond entertainment.

For our research, we have integrated game elements into various courses to foster transparent assessment, let students know their achievements, clarify our expectations, and enhance motivation. We will offer a deep overview of motivation and show different gamified environments developed with or without technology.

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mathematical TRANSGRESSIONS

Guillermo Bautista, Jr., Abigail Gonzales, Theodosia Prodromou, Zsolt Lavicza, Markus Hohenwarter

Johannes Kepler University, Austria

Student Strategies in Function Art: Exploring Interdisciplinary Approaches

This study delves into function art creation among 235 students spanning Grades 8 to 12 in the Philippines. Function art is defined as "art constructions whose components include graphs or parts/segments of graphs of mathematical functions." This study aims to investigate the functions utilized, strategies employed, and the potential of function art in fostering transdisciplinarity within STEAM education. Employing content analysis, the study examines how students strategically utilize functions and other mathematical elements in their artwork.

The findings revealed that linear, quadratic, and sine functions emerge as preferred choices due to their versatility and practical utility. Furthermore, the study elucidates various strategies adopted by students in leveraging functions effectively. Despite its inherent transdisciplinarity, function art emerges as a platform that empowers students to navigate across disciplinary boundaries. It offers a variety of solutions that cater to distinct disciplinary domains.

This study underscores the potential of function art not only as a pedagogical tool but also as a catalyst for promoting transdisciplinary learning experiences within the STEAM landscape.

Filiz Mumcu, David Hornsby, Zsolt Lavicza, Chronis Kynigos

Johannes Kepler University, Austria

Enhancing STEM Education through the Fusion of Computational Thinking and Computer Science Education

In contemporary discourse on computer science (CS) education, the focus has shifted from solely training experts to advocating for its accessibility to all learners, spurred by Wing's influential study in 2006. This study emphasized the importance of computational thinking (CT), a concept popularized by Seymour Papert (1980), who also developed the visual programming language Logo. Papert's notion, grounded in Piaget's theories of cognitive development, advocates for early exposure to computer programming, underlining the significance of CT from an early age. Despite early recognition of the potential of CS education to foster interdisciplinary thinking, it wasn't until Wing's work that CT gained significant traction. Integrating CT into various disciplines democratizes CS education, impacting learners' cognitive, social, and emotional development.

Integrating CS education into the broader framework of STEM (Science, Technology, Engineering, and Mathematics) is crucial for fostering interdisciplinary learning and honing problem-solving acumen.

Within STEM education, the infusion of CS not only enriches the learning environment but also cultivates synergies between different domains, yielding substantial benefits.

Within the realm of STEAM education, CS education assumes significant importance, with CT at its core. But what exactly does computational thinking entail? Lodi (2020, p. 120) emphasizes that computational thinking is a way of thinking or a thought process for problem-solving and that the formulation and solution of the problem should be expressed in a way that allows an "external" processing tool (a human or a machine) to perform it. CT is a thinking process involving problem decomposition, pattern recognition, abstraction, and algorithmic design.

The symbiotic relationship between STEM and CT underscores the intrinsic connection between problem-solving and technological understanding. While CT forms a cornerstone of CS education within STEM, the emphasis has traditionally leaned towards programming skills. CT is facilitated by computational practices such as sequences, loops, parallelism, events, conditionals, operators, and data handling—essential elements for programming and problem-solving in CS. So, integrating CT into STEAM education necessitates appropriate tools and methodologies. Block-based programming offers a user-friendly and visually intuitive approach to teaching programming concepts, making it accessible to students of all ages and backgrounds. Platforms like Scratch and Blockly provide environments where students can drag and drop blocks to create programs, thereby reducing the initial barriers to learning programming languages. Unplugged computer activities offer a pragmatic solution, particularly for educators lacking a background in CS or programming, providing hands-on experiences without reliance on computers, thus mitigating barriers such as cost and technical constraints.

The concept of CS unplugged originated from the endeavour to make CS and mathematics education more engaging and accessible. CS unplugged activities, pioneered by Bell, Witten, and Fellows, entail teaching CS or CT concepts sans computers, emphasizing practical experiences and a constructivist learning approach, wherein students explore concepts through hands-on activities. Research underscores the effectiveness of the CS unplugged approach in introducing programming concepts and algorithms before transitioning to computer-based practice. However, there remains a need for deeper pedagogical insights into how precisely CS education harmonizes within the broader STEM landscape.

In summation, the integration of CT into STEM education through CS education, block-based programming, and unplugged activities harbour immense potential for revolutionizing learning experiences. By equipping students with vital technological skills and nurturing creative problem-solving abilities, educators can prepare them to navigate the challenges of the future adeptly. Moreover, by empowering teachers to embrace innovative pedagogical paradigms, educational institutions can ensure that STEM education remains pertinent and impactful in an increasingly technology-driven world.

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June 4, 2024

Lecture 3 (15:00 – 16:00)

Zsolt Lavicza

Johannes Kepler University, Austria

Inspiring teachers' classroom innovations through the integration of Augmented/Virtual Reality and 3D printing into their practices

The swift evolution of 3D technologies has opened up diverse opportunities for 3D modelling to be utilised in education both in digital and physical formats. As industries like medicine, construction, and technology design increasingly rely on 3D modelling, its potential applications in education are increasingly gaining traction. This talk, based on studies conducted by the STEAM education research group at the Linz School of Education, Johannes Kepler University, Austria, delves into introducing Augmented/Virtual Reality and 3D printing in teacher education across various countries. We explored teachers' perceptions, established the requisite educational ecosystem for 3D technologies, evaluated pedagogical approaches for integrating 3D modelling into classrooms, and emphasised the incorporation of arts and culture to inspire students. Our initiatives extend to creating 3D resources for students with disabilities and those from disadvantaged communities, as well as fostering girls' engagement in STEM studies through 3D modelling. The core objective of our studies is to empower teachers and students as innovators in utilizing these novel technologies. Additionally, we addressed the demand for new theoretical and methodological approaches by expanding our work from mathematics to STEAM, introducing a STEAM+X approach, and supplementing Design Based Research (DBR) with User Experience (UX) research methodologies to adapt to rapid technological changes. In this talk, exemplary practices will be described showcasing secondary and primary education in Europe, Asia, Africa, and Latin America.

June 4, 2024

Lecture 4 (16:00 – 17:00)

James Tanton

Mathematical Association of America in Washington D.C.

The astounding visual story of place value: from base 10 to base x

It's a global phenomenon in mathematics! Over 8 million students and teachers from over 170 countries and territories across the planet have, with a single visual model, re-envisioned mathematics they thought they knew so well and embraced it in stunning new light. Early-school mathematics and high-school mathematics (and beyond!) are united in one beautiful and accessible whole.

Let me share this mind-blowing approach to mathematics with you too and see how standard school content really can serve as a portal to human joy, wonder, and awe.

June 4, 2024

Session 5 (18:00 – 20:00)

Tamara Díaz-Chang

Universidad Austral de Chile, Chile

Metacognitive strategies as cognitive transgressions in the learning process of certain mathematical concepts al the university level

The primary aim of this study is to examine how students' cognitive transgressions can be identified in the recognition processes of unconscious cognitive mechanisms appearing in the learning of some mathematical concepts related to the convergence of sequences at the university level. The data analysed was collected from the cognitive activity of three undergraduate students who voluntarily participated in the study. The implemented methodology was supported by a qualitative approach. The study occurred in two phases: in the first phase, students answered some questionnaires, and in the second phase, stimulated recall interviews involving the verbalization of the cognitive processes retrospectively, were conducted. The analysis shows metacognitive strategies through which students' unconscious misconceptions were examined, through self-reflection on previously acquired mental models or schemas. It also shows how such cognitive transgressions could be a result of a conscious or semiconscious activity, although it may be not intentional or not anticipated, and just a resolution of the cognitive conflict.

Ángel Homero Flores Samaniego

Universidad Nacional Autónoma de México

Neuroscience and didactics: a dialogue in construction?

In recent times, due to the advances in the study of human brain and its physiology, there has been a growing interest in figure out the physiological mechanisms behind human cognition in what is getting to know as cognitive neuroscience. Now, if cognitive neuroscience studies brain mechanisms and its functionality involved in learning, it is but logical to ask for connections between this science and education.

De Smedt, et. al. (2011) argued that the relationship between neuroscience and education should be a "two-way street" with many bi-directional interactions between them. If we review the academic literature in Journals in both fields we will notice that this two-way street is more a one direction street: from cognitive neuroscience to education.

Besides, researchers in both fields speak about education research a field, in my opinion, related more to a pedagogical level (id est, research on educational phenomena); but what about didactics (the study of learning in a classroom environment)? It is necessary a dialogue between cognitive neuroscience and didactics? There would be mutual benefits in such a dialogue?

My experience as mathematics teacher and researcher tells that could be useful contributions from didactics to cognitive neuroscience and vice versa. But, to start the construction of such a dialogue we need to change the conception of didactics and the model of brain that tends to be built from cognitive neuroscience outcomes.

I will outline a proposal that could be a starting point on the construction of the dialogue between neuroscience and didactics.

Jean-François Maheux

Université du Québec à Montréal, Canada

Exploring the Transgressive Organics of Mathematical Traces

On that day, Mary said to a group of middle schoolers, “I found a new way to reduce fractions,” and continued while writing on the whiteboard: “Here, I have 16 over 64, and if I cross out the 6s, I get one fourth. Is this method working?”. Students swiftly reacted, and the next 20 minutes were dedicated to the examination of cases of fractions for which invalid reduction (Fried & Goldberg, 2010) works or not, and perhaps to explain why. The unusualness of what we now call weird fractions (Stuffelbeam, 2013) and what took place on that day helps highlight the ambiguous nature of traces as conceptualized by Derrida (2001), for example (also see Maheux, 2023). By the micro-analysis (Phillips & Hardy, 2002) of the observed interactions around 16/64=1/4, we can explore the transgressive organics of mathematical traces, detailing how traces transgress activity and vice versa. This view also account for how mathematical ideas propagate across temporal, spatial, social, cultural (etc.) boundaries. It formulates a perspective on transgression as “illegitimately and impossibly going beyond,” which also describes how students can become familiar with (new) mathematics despite the infamous “learning paradox” (Bereiter, 1985).

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mathematical TRANSGRESSIONS

Jean-François Maheux

Université du Québec à Montréal, Canada

Simon Théberge

Université du Québec en Abitibi-Témiscamingue, Canada

Exploring Connections Between Philosophy for Children and Relationships to Mathematical Knowledges in Educational Practice

Transgressions in mathematics education means for us departing from dominant approaches focused solely on performance (Glatthorn, 1993; Boutin, 2004; Demers, 2016). In 2018, Maheux & Proulx emphasized the importance of shifting towards "doing mathematics" rather than teaching or learning it. Building upon their work, this study suggests incorporating philosophical activities into mathematics education as a means to enhance students' experiences.

Philosophy for children adapted to mathematics (e.g., Lafortune et al., 2003;) extends beyond the mere transmission of mathematical knowledge. It encourages students to delve into their feelings, emotions, concepts of self, and epistemological beliefs to fostering reflective and creative thinking. This shift challenges traditional paradigms in mathematics education, transgressing from the notion of merely enhancing students' performance. Charlot's (1997) perspective helps us appreciate it in terms of developing rich relationships to knowledge. Integrating philosophical activities into mathematics education offers a pathway to explore how such practices can contribute to students mathematical education fostering both an engagement with mathematical ideas, and a nuanced and multifaceted relationship with mathematics itself.

This presentation will delve into our ongoing project, exploring how these connections between philosophy and relationships to knowledge can transgress traditional boundaries in mathematics education, fostering a deeper understanding of the dynamic interplay between philosophical inquiry and mathematic education.

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Important notes:

- Speech times are given in Polish time (CEST).
- Each session presentation is given 30 minutes at the speakers' disposal (including time for questions).
- All lectures and presentations will be delivered online.
- A link that allows participation in the conference will be sent to all registered participants at the end of May.
- Since the conference is attended by people living in different time zones, **each talk will be recorded**, and participants will receive one-week access to all recordings.